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NISOD INNOVATION ABSTRACTS

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IMAGINE THAT!

Immanuel Kant proposed that mathematics requires two things: imagination and rigorous logic. Instructors of college numerical math (from here on "numerical math" refers to those college courses traditionally labeled developmental or remedial, including pre-algebra) tend not to get much chance to use imagination and rigorous logic because the basics demand a lot of time. Unlearning shards of mathematical information sometimes has to precede real learning for students in these classes.

A wide repertoire of methods is needed to get these students to overcome under-developed skills and heightened anxiety. And this requires some imagination, which is present in some texts and practices. Discussing how to teach various topics with colleagues leads to some interesting and imaginative areas—some of which result in nothing more than comic relief. However, we do still tend to avoid the rigorous logic, but we do *teach* "logic." After all, we have to attempt to make sense of the material through which students have suffered previously. But, as I see it, the isolation of the individual operations, algorithms, "rules," etc., has to be overcome. Students are looking for a sense of order in all this stuff. They sincerely want to learn and understand; they almost demand it, although in most cases it is a very quiet shout-out. Does all this stuff fit together, or is it just a heap of disconnected ideas and operations? Do students not "get it" because of too much or too little imagination on their parts, or perhaps ours?

I have learned to listen carefully to the silence and observe closely the slouches, grimaces, and ceiling stares. I have learned, by probing and prodding and wrestling with students' anxieties, that there are some remarkably imaginative (although most times inaccurate) perspectives operating. Students will ask about patterns they see; they ask if they can do this or that as a shortcut. Many "what if" questions pop up. And these questions cry "help me." I believe that part of our role as instructors who are using our imaginations is to allow the imaginations of students to thrive. It requires funneling, pruning, redirection, questioning, and patience to allow students to explore an idea and find it useful or not, but this establishes your instructor role as one in which you can present nontraditional approaches. In many instances, students who have been stymied by the traditional are very responsive to the nontraditional.

I present one example and admit that although it may not be successful for all students, it is for most. I receive feedback, and it is generally positive. But more critically, I discuss with students that the procedure came from a higher mathematics and that what they are studying can and will be used in later math classes. I put it into a context that demonstrates the continuity of the idea—something way out in one's math future provides a framework for something basic and can be used in the next level math course. This context, not only the procedure, was a motivational learning tool.

The example is PEMDAS (Parentheses, Exponents, Multiplication, Division, Addition, Subtraction)—the traditional acronym for order of operations—which does not seem to work for all students. There are alternative acronyms. But what came to mind, as I was preparing to present order of operations, was my *not being excited* by it. What? Not excited? So why should students be excited? I "meditated" on this for awhile; several thoughts emerged, and I packaged them for presentation. So, students imagining the many ways (some correct, most incorrect) that PEMDAS can be interpreted seemed to have awakened my imagination.

PEMDAS is a directive that is easy to recall but not easy to follow. There are holes. For example, "P" really means work inside the parentheses. Be careful because parentheses also signal multiplication. "E" also includes "R"—roots. The "MD" is sometimes a muddle because, although the M comes before the D, this is not the order of the operations. When seen in a problem, do them left to right. Does that mean all multiplication left to right before all divisions, left to right, or all multiplication and division left to right? And, the left to right works for "AS" too—except, as students learn later, there is this thing called the commutative law which allows them to ignore left to right. But when is it okay? What to do, what to do?

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My first "aha" moment was that PEMDAS is applicable when solving equations, but it is rarely mentioned as a procedure. When I eventually presented what turned out to be a non-traditional approach to doing order of operations, I showed the class how it worked in basic math and in algebra. I did this even though equations are not part of the numerical math curriculum, and this thought got me connected to another algebraic concept.

My second "aha" moment was realizing that there is a very traditional algebraic principle which would seem to be a non-traditional method for dealing with order of operations in numerical math classes. Identifying terms in a polynomial is done by noting that the plus sign and the minus sign separate (or if you prefer, connect) terms. So, a polynomial expression like $3x^3 – 2x^2 + 4x(x – 1) – 5$ has four terms. What also must be mentioned is not to include plus or minus signs inside grouping symbols. But the same identification process can be used inside grouping symbols. Polynomial work is not done in numerical math, just as solving equations is not done in numerical math, but thinking about these "higher order" operations brought about a new approach for working in numerical math.

Consider a problem involving order of operations, like 4•3 –15 +2(4 +1). Rather than using PEMDAS, the student first identifies the terms in this expression by starting to underline from left to right until hitting a plus or minus sign. Then lift the pencil past the sign, start underlining again, and continue doing this to the end of the expression. The result will be $\underline{4•3}$ – $\underline{15}$ +$\underline{2(4 +1)}$. The point is to identify the terms and then do all the indicated operations within each term until you get a single number. Simplify each term before doing operations between terms. It emphasizes that the 15 is not to be subtracted from the 3 nor is the 2 to be added to the 15. The 3 is to be multiplied by the 4, and the 2 is to multiply the result of the operation inside the parentheses. Then what is seen is 12 –15 +10. Another aspect of this method is that if the student "reads" the positive and negative signs as *signs of numbers, not signs of operations*, it can be demonstrated that the operation is "add all these numbers." Reading it this way allows using the commutative property rather than using the "left to right" addition/subtraction PEMDAS strategy. I drew on some of the other material already learned and showed how it worked to finish the problem.

This same strategy works in solving equations. Given the equation 2 + 3 (x – 4) = 5 – 2(x + 1) and asked to solve it, the first step is to identify terms on both sides of the equation. Using the underlining to identify terms; the result would be $\underline{2} + \underline{3 (x – 4)} = \underline{5} – \underline{2(x + 1)}$, which again clarifies what needs to be done. On the left side, the 2 does not get added to the 3 because they are separate terms; and on the right side, the 2 does not get subtracted from the 5 because they are separate terms. Each term has to be simplified completely by doing the indicated operations before the plus and minus signs are "used." I demonstrated this in class, but we did not solve equations, only identified the terms and what needed to be done to "simplify" each.

So, in principle, when students are learning and employing this procedure for order of operations in numerical math, they are learning a first step in solving equations (when "simplifying" is needed) and also learning to identify terms in polynomials.

But more critically, in my estimation, is that students saw how a basic concept is not something to learn and discard but rather to be used throughout all levels of their math experience. In a sense, the arithmetical algebra and the symbolic algebra are the same in that the operations are consistent within and between these domains.

The unsolicited feedback from students—both directly (by their telling me) and indirectly (by their mastering the method)—supports Kant's comment about using imagination in math. I commented to the students that their imaginations—perhaps sometimes mathematically misguided—prompted my own. This seemed to reinforce their learning and their wanting to learn and explore more ideas.

I believe it is the instructor's responsibility, particularly in developmental/remedial and pre-algebra classes, to engage students in being aware of these basics that knit the various levels of math together. I believe this because it is not only the skills that must be learned but the context for applying the skills. In essence, that is why basics are the basics; they are fundamental and necessary to the entire mathematical enterprise. And, knowing that these skills have importance to later learning puts more "reality" into learning them. Further, knowing that they can play a little with some of the rules helps students understand how the rules work—imagine that!

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